Laboratory Work No. 4.
Determination of Beta Decay Electron Range by Absorption Method

The purpose of this work is to study the mechanism of electron interaction with matter. Using the absorption method, the maximum range of electrons formed during β decay is determined.

Contents

1. Introduction

When passing through a substance, any charged particle interacts with the atoms of that substance. The registration of particles also occurs as a result of their interaction with the detector substance. The interaction of particles with matter depends on their type, charge, mass, and energy. Charged particles ionize the atoms of matter by interacting with the electrons in the atoms. The interaction of particles depends on such characteristics of a substance as density, atomic number, and the average ionization potential of the substance.

Each interaction leads to a loss of energy of the particle and a change in its trajectory. In the case of a beam of charged particles with kinetic energy T passing through a layer of matter, their energy decreases as the substance passes through, the energy spread increases, and the beam expands due to multiple scattering. Various reactions can occur between a particle passing through the medium and particles of matter (electrons, atomic nuclei). As a rule, their probability is noticeably less than the probability of ionization. However, reactions must be considered in cases where the particle interacting with the substance is neutral. For example, neutrinos can be detected only by their interaction with the electrons of atoms or the nucleons of the nuclei of the detector's substance. And neutrons are detected by recoil protons or by the nuclear reactions they cause.

2. Beta decay

The phenomenon of β decay consists in the fact that nucleus (A, Z) spontaneously emits leptons of the first generation — an electron/positron and an electron antineutrino/electron neutrino, turning into a nucleus with the same mass number A, but with atomic number Z increased or decreased by 1 $(Z \pm 1)$. During electron capture, the nucleus absorbs one of the electrons of the atomic shell (usually from the K-shell or L-shell closest to it), emitting an electron neutrino. There are three types of β decay: β^- decay, β^+ decay and ε-capture:

$$
\begin{aligned}
\beta^-: &\quad (A, Z) \to (A, Z+1) + e^- + \bar{\nu}_e, \\
\beta^+: &\quad (A, Z) \to (A, Z-1) + e^+ + \nu_e, \\
\text{e}: &\quad (A, Z) + e^- \to (A, Z-1) + \nu_e.
\end{aligned}
$$

β decay is governed the weak interaction, one of the four fundamental interactions. β decay is not an intra-nuclear process, but an intra-nucleonic one. That is, a single nucleon decays inside the nucleus via the following processes:

$$\begin{aligned}
\beta^-\!: &\quad n \to p + e^- + \bar{\nu}_e, \\
\beta^+\!: &\quad p \to n + e^+ + \nu_e, \\
\mathrm{e}: &\quad p + e^- \to n + \nu_e.
\end{aligned}$$

Like any kind of decay, β decay can only take place when energetically favourable conditions are met. That is, if we neglect the mass of the neutrino (in practice, $m_\nu < 0.7$ eV), it is imperative that

$$\begin{aligned}
&M(A,Z) > M(A,Z+1) + m_e, \\
&M(A,Z) > M(A,Z-1) + m_e, \\
&M(A,Z) + m_e > M(A,Z-1).
\end{aligned}$$

where $M(A,Z)$ is the mass of the nucleus (A,Z) and m_e is the mass of the electron. Essentially, the mass of the initial products should be greater than the mass of the final products.

The decay of a free neutron $n \to p + e^- + \bar{\nu}_e$ also refers to β decay. The average lifetime of a neutron is $\tau = 878.4 \pm 0.5$ s. The mass of a neutron is greater than the sum of the masses of a proton and an electron, which determines the possibility of its spontaneous decay. A free proton cannot undergo the β^+ decay $p \to n + e^+ + \nu_e$, since a proton is lighter than a neutron. However, inside the β^+ active nucleus, such a process is possible as a result of the interaction between the nucleons, as these processes take into account the energy of the nuclei.

β radioactive nuclei are the most numerous type of radioactive decays (see Fig. 1), located almost in the entire range of values of the mass number A, starting from unity (free neutron).

Consider the energy and momentum conservation laws for the process of

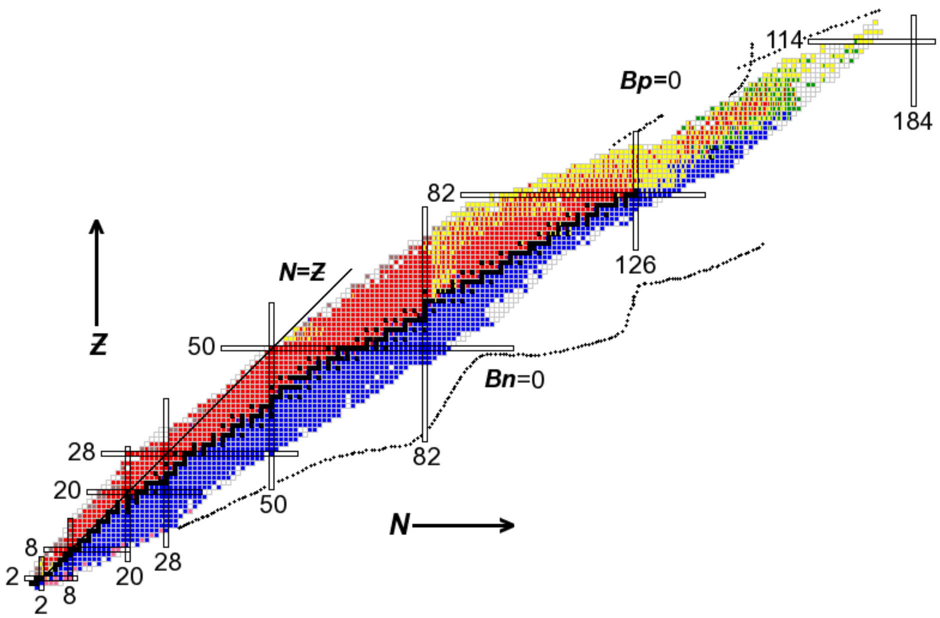

Figure 1. N-Z diagram of atomic nuclei. Red and blue colors are beta radioactive nuclei.

$\beta^{\pm}$ decay:

$$M_i c^2 = M_f c^2 + m_e c^2 + T_f + T_e + E_\nu, \tag{1}$$

$$\vec{p}_i = \vec{p}_f + \vec{p}_e + \vec{p}_\nu. \tag{2}$$

Here, M and m stand for the mass of the nuclei and particles, respectively, T is the kinetic energy, and $E = T + mc^2$ is the total energy, $\vec{p}$ is the momentum and indices i and f refer to the mother (initial) and daughter (final) nuclei. Let us denote the mass difference of the initial and final decay participants as Q. This value, called decay energy, carries the meaning of the energy released during the decay. As follows from (1), the decay energy essentially is distributed between the kinetic energies of the products of the decay. In the case of β decay, the mass of the daughter nucleus is by several orders of magnitude greater than that of the two leptons, which results in the kinetic energy T_f being negligible compared to T_e and T_ν. Thus, the energy Q is predominantly distributed between T_e and T_ν, meaning the maximum kinetic energy of both leptons is equal to decay energy Q. For various nuclei, this value ranges from several eV to around 15 MeV.

For β^- and β^+ decay, the energy spectrum of the products is continuous (see Fig. 2). This follows from the fact that there are three products, aka three kinematic characteristics bound by two constraints (two conservation laws) that cannot be resolved in a unique way, and thus take on values from a continuous range. The spectrum of each three-particle decay product has a so-called "upper bound" — the maximum kinetic energy value. It corresponds to the kinematic situation when a given particle moves in the opposite direction relative to the other two. In contrast, the energy spectrum of ε-capture is discrete, as their are only two products in this decay.

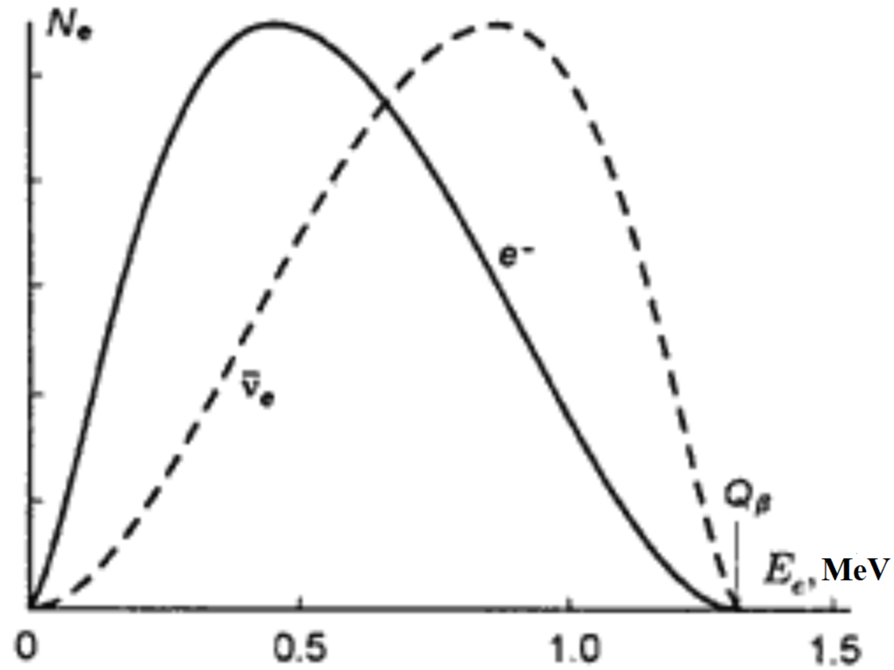

Figure 2. Spectra of electrons and antineutrinos formed during β decay of isotope ^{40}K.

What determines the probability of the decay? It depends on the decay energy, the initial and final states of the nuclei, and other factors. Sargent rule is a rule that relates the probability of beta decay to its energy. It states that at high energy releases, the rate of β decay depends on the energy to the fifth degree.

$$\lambda \sim Q^5$$

During β^+ decay and electron capture, the same process of proton-neutron conversion occurs in the nucleus. Therefore, both of these processes can run in the same isotope and often compete with each other. From an energy point of view, electronic capture is more advantageous. In particular, if the initial and final nuclei satisfy the inequalities:

$$M(A, Z-1) + m_e > M(A, Z) > M(A, Z-1) - m_e,$$

then electron capture is allowed, but β^+ decay is impossible. This situation occurs during the conversion of beryllium isotope ^{7}Be into lithium isotope ^{7}Li. This nucleus undergoes electron capture ($^7\mathrm{Be} + e^- \to {}^7\mathrm{Li} + \nu_e$) but not β^+ decay, since the mass difference of the nuclei is around 0.862 MeV, which is less than $2m_e c^2 = 1.022$ MeV.

An important factor influencing the probability of decay is the orbital momentum carried away by the lepton pair. In quantum mechanics, angular momenta of two types are differentiated. The orbital angular momentum is defined in a way similar to the classical angular momentum, albeit taking on only a set of discrete (integer) values. It describes the angular momentum of relative motion of several particles. In contrast, the kind of angular momentum known as the spin corresponds to an inherent and unchanging internal rotational state of the particle, although this rotational state cannot be interpreted classically as rotation of a body around its own axis. Spins can also take on only discrete values. The sum of orbital angular momenta and spins of all particles, known as total angular momentum, is conserved in any given process.

In the case of β decay, the factor of the orbital lepton angular momentum has a greater impact on the decay probability when compared to the influence of the energy conditions. Let us consider the impact of these factors on the example of isotope ^{60}Co (see Fig. 3).

From the energy point view, ^{60}Co into a number of states of ^{60}Ni, namely:

- the ground state of the final nucleus with spin (in units of Planck constants $\hbar$) $J = 0$,

- the first excited state with $J = 2$,

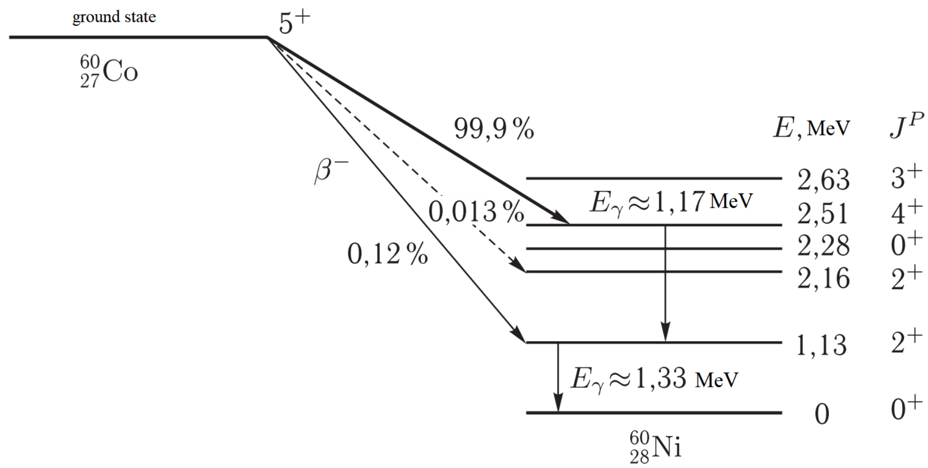

Figure 3. Beta decay of ^{60}Co.

- the higher energy states (2.16, 2.28, and 2.51 MeV) with spins 2, 0, and 4.

The first channel is most energetically advantageous, meaning the largest amount of energy Q is released in this process. However, in reality, almost 100% of β transitions occur along the least energetically favorable path among the above, that is, to the fourth excited level of ^{60}Ni with a spin of 4.

To understand the reason why this particular decay channel is by far the most probable, consider the law of conservation of angular momentum in the β decay of ^{60}Co:

$$^{60}_{27}\mathrm{Co} \to {}^{60}_{28}\mathrm{Ni} + e^- + \overline{\nu}_e,$$

$$\vec{J}_{\mathrm{Co}} = \vec{J}_{\mathrm{Ni}} + \vec{s}_e + \vec{s}_\nu + \vec{l}_{e+\nu}.$$

Here $l_{e+\nu}$ is the sum of the orbital momenta carried away by the leptons, s_i is the spin of particle i and J_X is the spin of nucleus X. Let us write down the law of conservation of angular momentum for the aforementioned β decay channels of each channel and find the possible values of $l_{e+\nu}$. Due to quantum uncertainties, quantum vectors of angular momenta can add up in different ways, with their lower and upper boundaries dictated by triangle inequality rule:

$$J_i - J_f \leq |\vec{J}_i + \vec{J}_f| \leq J_i + J_f.$$

Together with the conservation law this gives

$$J_i - J_f \leq |\vec{s}_e + \vec{s}_v + \vec{l}_{e+\nu}| \leq J_i + J_f.$$

Spins of particles are inherent and always fixed values, and for the electron and neutrino they are equal to $\frac{1}{2}$. Furthermore, we recall that quantum angular momenta only take on discrete values, which results in the following relations:

$$J_f = 0: \quad \vec{5} = \vec{0} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \vec{l}_{e+\nu} \Rightarrow \vec{l}_{e+\nu} = 4, 5, 6; \tag{3}$$

$$J_f = 2: \quad \vec{5} = \vec{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \vec{l}_{e+\nu} \Rightarrow \vec{l}_{e+\nu} = 2, 3, 4, 5, 6, 7, 8; \tag{4}$$

$$J_f = 4: \quad \vec{5} = \vec{4} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \vec{l}_{e+\nu} \Rightarrow \vec{l}_{e+\nu} = 0, 1, 2, \ldots, 9, 10. \tag{5}$$

The application of angular momentum conservation law to the listed decay channels of ^{60}Co shows that only in the case of β decay to an excited level with spin 4, the orbital momentum carried away by an electron and a neutrino can be zero. This is the so-called "allowed" transition. They are carried out with almost 100% probability, although it is the least energetically advantageous of all the listed decay channels. Note that the effect of the transition energy on the probability of β decay is clearly manifested when comparing the probabilities of two transitions of the same spin: $5 \to 2$ (2.16 MeV) and $5 \to 2$ (1.33 MeV). The second one proceeds with a higher energy release and therefore occurs with a higher (almost an order of magnitude) probability.

Under equal conditions, the ratio of the probabilities of decays with $l_{e+\nu} = 0$ (W_0) and $l_{e+\nu} \neq 0$ (W_l) is given by:

$$W_{l_{e+\nu}}/W_0 \simeq (R/\lambda)^{2l_{e+\nu}},$$

where R is the radius of the nucleus, and λ is the wavelength.

3. Interaction of electrons and positrons with matter

When passing through matter, electrons lose their energy due to electromagnetic interactions with the electrons and atomic nuclei of the absorber. For electrons with relatively low energy (below the so-called critical energy), as well as for heavy charged particles, the energy losses are dominated by ionisation and excitation of the absorber's atoms — the ionisation losses. At electron energies exceeding the critical energy, another interaction mechanism becomes dominant: the emission of electromagnetic radiation due to deceleration — radiative losses, otherwise known as bremsstrahlung.

According to classical electrodynamics, a charge undergoing acceleration a radiates energy. The radiated power W is given by

$$W = \frac{2}{3}\frac{e^2 a^2}{c^3}.$$

The acceleration of a particle with charge ze and mass m passing at an impact parameter b from an atomic nucleus of charge Ze can be estimated as

$$a \approx \frac{Zze^2}{mb^2}.$$

The acceleration in the field of atomic nuclei is proportional to the product of the nucleus charge and the particle charge and is inversely proportional to the particle mass. Therefore, for example, the energy radiated by a proton when decelerated is smaller than that radiated by an electron in the same stopping field by a factor of order $\sim 3.5 \times 10^6$. For this reason radiative losses, which are important for high-energy electrons, are negligible for heavy charged particles passing through matter.

For electrons the specific (i.e., per unit length) radiative losses grow with electron energy and are proportional to the square of the absorber nuclear charge. The energy at which the specific radiative losses become equal to the specific ionisation losses is called the *critical energy*. Table 1 shows that critical energies for light absorbers are tens of MeV, i.e. much higher than typical β decay electron energies. Thus, for β decay electrons in light absorbers such as aluminium, energy losses are dominated by ionisation.

Table 1. Critical energies for several materials

Material	T_{crit} (MeV)
H (liquid)	278.02
Air	87.92
Ar (liquid)	32.84
C (graphite)	81.74
Al	42.7
Fe	21.68
Cu	19.42
Pb	7.43

The magnitude of ionisation energy loss per unit path length is described by the Bethe-like expression (in a simplified form):

$$-\left(\frac{dT}{dx}\right)_{\mathrm{ion}} = \frac{4\pi e^4 z_1^2}{m_e v^2} ZNB, \tag{6}$$

where $z_1 e$ is the particle charge, v its velocity, N the number density of atoms in the absorber (atoms per cm^3), Z the atomic number of the absorber, m_e the electron mass, and B is a slowly varying logarithmic function (the stopping number) which depends on velocity, the mean ionisation potential of the absorber atoms, and the reduced mass of the interacting particles. In the non-relativistic energy range the influence of the logarithmic factor B on the energy dependence of ionisation losses is weak, so that roughly

$$-\left(\frac{dT}{dx}\right)_{\mathrm{ion}} \sim \frac{1}{v^2}.$$

The dependence of ionisation losses on the absorber material is determined by the product ZN (and weakly by the mean ionisation potential). It is convenient to introduce the specific (mass stopping) power, i.e. the loss per unit areal density $x\rho$ (g/cm^2), where ρ is the absorber density. Using the

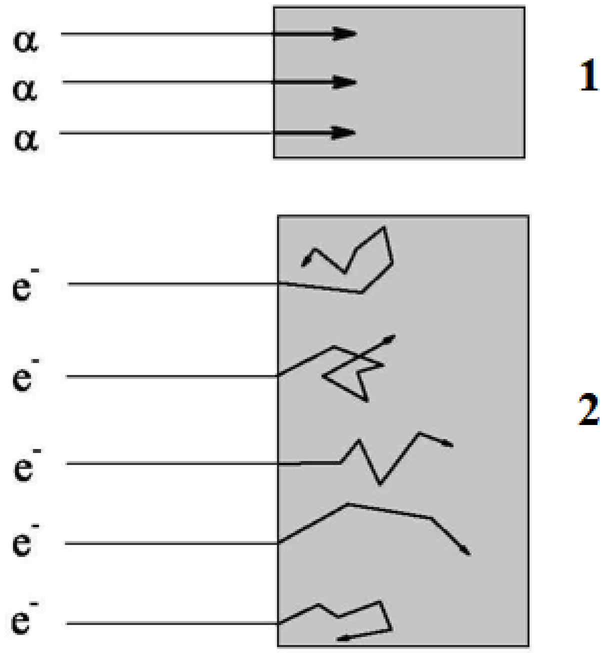

Figure 4. Schematic behaviour of initially parallel beams of relativistic particles in matter: (a) α-particles, (b) electrons.

relation

$$\frac{ZN}{\rho} = \frac{N_A \rho Z}{A},$$

with Avogadro's number N_A, one can express stopping powers in units independent of density.

The *range* R of a charged particle is the path length over which its initial kinetic energy T_0 is dissipated by interactions with the medium:

$$R = \int_0^{T_0} \left(-\frac{dT}{dx} \right)^{-1} dT,$$

either expressed as a length (cm) or as mass thickness (g/cm^2), where $R[\mathrm{g/cm}^2] = R[\mathrm{cm}] \cdot \rho[\mathrm{g/cm}^3]$. Thus the range is a function of the particle kinetic energy; measuring ranges allows determination of particle energies.

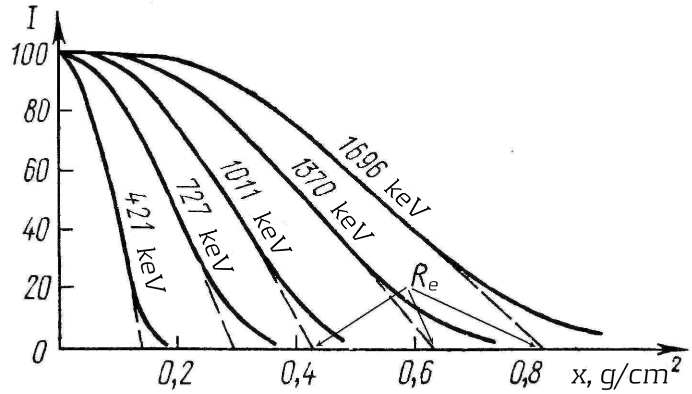

Figure 5. Dependence of transmitted intensity I of an initially monoenergetic electron beam on the thickness of an aluminium absorber for different beam energies. R_e denotes extrapolated range for monoenergetic electrons.

Multiple scattering of electrons

For heavy non-relativistic charged particles, due to their large mass and rather short range, the mean scattering angle is small and the trajectory is essentially straight. The small mass of electrons strongly influences their behaviour in matter. In collisions with atomic electrons and nuclei electrons are often scattered by large angles and follow a tortuous trajectory (see Fig. 4). Therefore for electrons the process of multiple scattering on atoms significantly affects their range.

Because of multiple scattering the direction of motion of an electron is substantially deviated from the initial one, and the total path length of the electron may exceed by a factor of 1.5–4 the projected range (the distance along the initial direction). For monoenergetic electron beams the transmission (intensity) as a function of absorber thickness shows plateaus at small traversed thickness for high energies (when $T \gg m_e c^2 = 511$ keV), since most electrons continue roughly in the initial direction; at larger thickness and lower energies the angular spread increases and the transmitted intensity decreases (see Fig. 5).

For electrons emitted in β decay (which, as discussed previously, feature a continuous energy spectrum), attenuation of the flux passing through matter occurs both because of scattering out of the forward direction and because lower-energy electrons are absorbed earlier. The absorption curve for such a

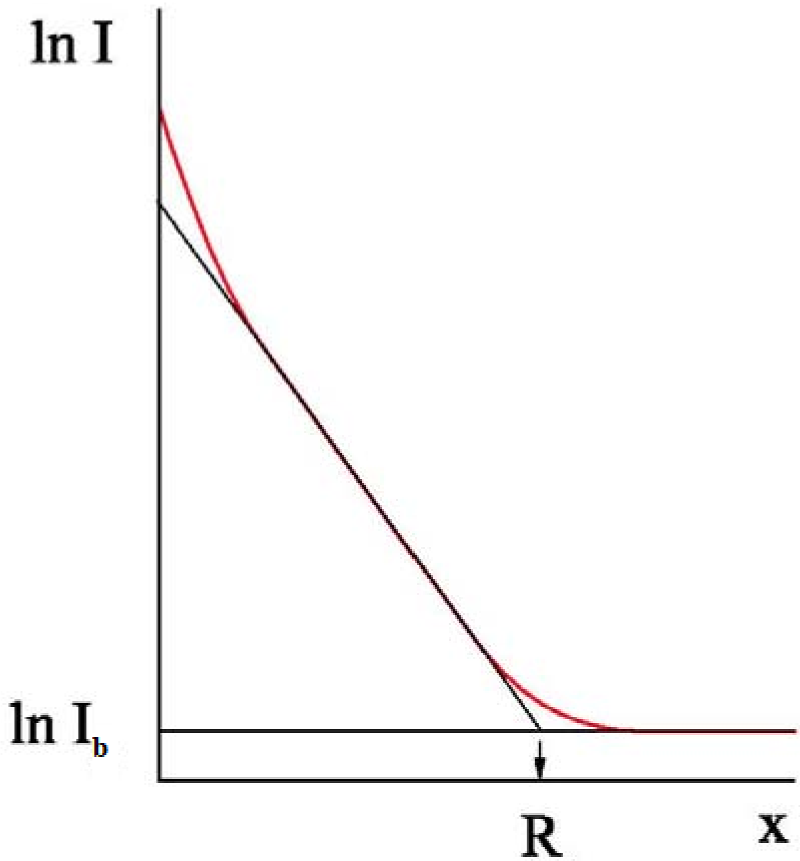

Figure 6. Dependence of $\ln I$ of electrons from a β-source on absorber thickness x. R is the extrapolated range.

source is the sum (integral) of monoenergetic curves and, therefore, lacks a simple linear region. However, plotting the total intensity $I = I_\beta + I_{bg}$ in a semi-logarithmic scale often reveals a region that can be approximated by a straight line. The *extrapolated range* R_e is defined as the absorber thickness at which the linear continuation of that region intersects the background radiation level $\ln I_{bg}$, which the registering equipment detects regardless of the experimental setup (see Fig. 6). Here, using the total intensity is convenient to avoid divergence of $\ln I_\beta$ when I_β becomes small.

Empirical formulae for the extrapolated range of electrons in aluminium as a function of kinetic energy T (MeV) are:

$$R(\mathrm{Al}) = \begin{cases} 0.4\,T^{1.4}, & T < 0.8 \text{ MeV}, \\ 0.54\,T - 0.133, & T > 0.8 \text{ MeV}, \end{cases} \tag{7}$$

where R is given in units of g/cm^2.

The extrapolated range in a material with atomic number Z and mass number A can be related to the range in aluminium by

$$R(A, Z) = R(\mathrm{Al})\,\frac{A}{Z}\left(\frac{Z}{A}\right)_{\mathrm{Al}}.$$

Note that the true path length of an electron in matter cannot be determined from the absorber thickness except for heavy particles which do not undergo significant Coulomb scattering. For electrons the trajectory is not straight; scattering effects are especially important in high-Z materials. In light materials scattering is weaker but still significant.

The number of electrons transmitted through an absorber of given thickness decreases gradually with thickness. The minimal absorber thickness that stops all β particles of initial energy T_β is sometimes called the *practical range* or *effective range*.

Extrapolated range and determination of $T_{\mathbf{max}}$

For determination of the maximum energy of a β spectrum by the absorption method, special nomograms can be used (shown and discussed in greater detail in section 4). Nomograms provide the relation between the thickness of aluminium corresponding to an intensity attenuation by a factor 2^n and the maximum energy $T_{\max}$. If the emitting nucleus has a charge $Z \neq 20$ or if the decay is positronic, a correction for the Coulomb interaction between the emitted charged particle and the daughter nucleus must be applied; the correction values are obtained from the corresponding graphs.

Interaction of positrons with matter

Positrons interact with matter according to the same relations as electrons. In addition, one has to account for annihilation of the slowing positron with an atomic electron; the annihilation cross section behaves roughly as $\sigma_{\mathrm{ann}} \sim 1/v$, so positrons annihilate after they have lost almost all their kinetic energy.

4. Experimental setup

The method of determining the extrapolated range of β decay electrons involves measuring the absorption curve of β radiation in a material. This is accomplished by measuring the dependence of the intensity of electrons I passing through a series of thin foils on the thickness of the absorber material. In this experiment, aluminium plates are utilized as the absorber.

The laboratory work begins with a thorough familiarization with the experimental setup. The setup includes:

- A radioactive source
- A Geiger counter for detecting electrons
- A preamplifier
- A counting device
- A power supply unit

The arrangement of the radioactive source, absorbers, and detector within the measurement unit is illustrated in Fig. 7.

5. Experimental Procedure

After completing the familiarization with the experimental setup, the measurement of the absorption curve for β decay electrons in aluminium begins. In addition to the electrons of the radioactive source, the counter registers

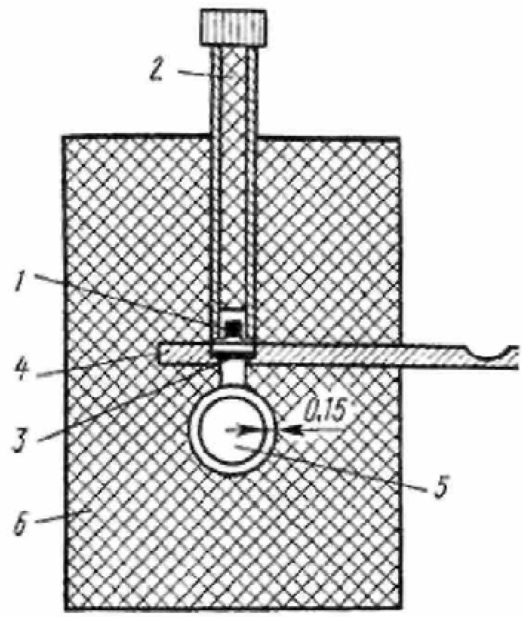

Figure 7. Diagram of the Geiger counter unit and the radioactive β source. 1 – radioactive source, 2 – holder of a radioactive source, 3 – absorbers, 4 – holders for absorbers, 5 – Geiger counter, 6 – lead casing.

the background formed by γ-quanta from the source, surrounding structures (walls, etc.) and cosmic rays that pass through the lead protection. The intensity of the β decay electrons I_β decreases with increasing thickness of the absorber x due to absorption and scattering according to an approximately exponential law. The intensity of the background I_{bg} is practically independent of the thickness of the absorber. Therefore, a linear section is observed in the semi-logarithmic dependence of the total intensity $I = I_\beta + I_{bg}$ on the thickness of the absorber in the region $I_\beta >> I_{bg}$. As the thickness of the absorber increases, the recorded intensity I tends to I_{bg}.

Exercise 1. Determination of the extrapolated range

To determine the extrapolated range, a graph is constructed showing the dependence of the logarithm of the intensity of registered electrons on the thickness of the absorber: $\ln I(x)$. The values of $\ln I$ are plotted with measurement errors as per Poisson distribution. The extrapolated range (R_e) is found by approximating the linear section of the $\ln I(x)$ graph with a straight line and extrapolating it to the background line. When determining the range, the thickness of the counter wall should be added to the value obtained from the extrapolation, as well as the average thickness of the film layer protecting the preparation from shedding. The effective range of electrons is taken as the extrapolated range. Finishing this exercise requires filling up the following table and plotting the $\ln I(x)$ graph.

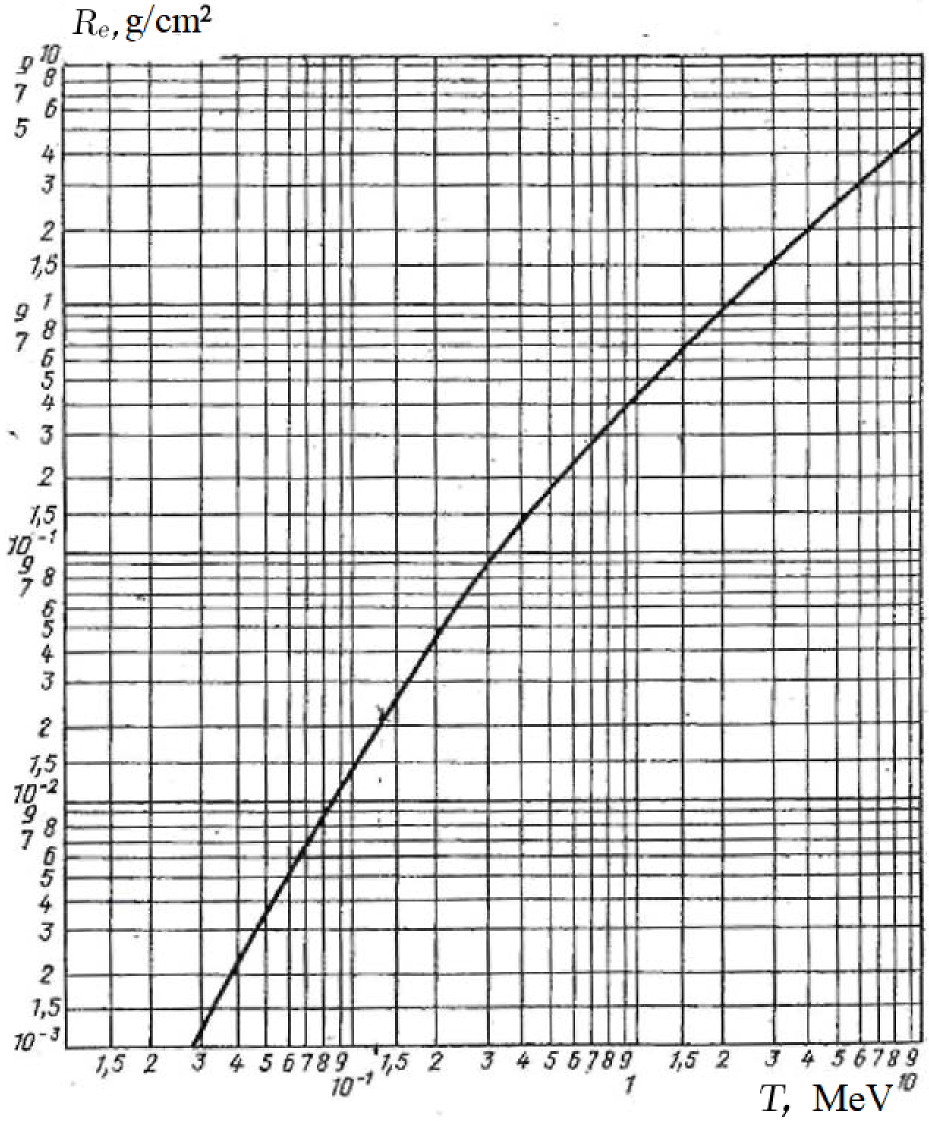

Figure 8. Dependence of the extrapolated path R of electrons in aluminium on their energy T corresponding to empirical relations (7).

Counts, N	Time, t (s)	Thickness, d (mm)	$I = N/t$ (s^{-1})	$ln(I)$

Exercise 2. Determination of the upper limit of the spectrum using the empirical relations and special nomograms

The results of the previous exercise can be used to determine the upper limit of the β decay spectrum in multiple ways. One such way (Method I) includes the use of empirical formulae (7) or their graph shown on Fig. 8. Method II involves using special diagrams known as nomograms, shown on Fig. 9.

The curves in Fig. 9 provide the dependence of the upper limit of the β spectrum T_{max} on the thickness of the absorber d_n, which decreases the intensity of the β particle beam by a factor of 2^n (2, 4, 8, 16, 32, etc.). The vertical axis shows the energy corresponding to the upper limit of the β spectrum, while the horizontal axis shows the thickness of aluminium. The value of thickness d for the lower curves is indicated at the bottom, for the upper curves —- at the top, and for the middle curves —- above the straight line drawn above the 4 MeV energy level. The indices on the curves ($n = 1, 2, 3, ...$) indicate the degree of intensity attenuation (by a factor of 2^n).

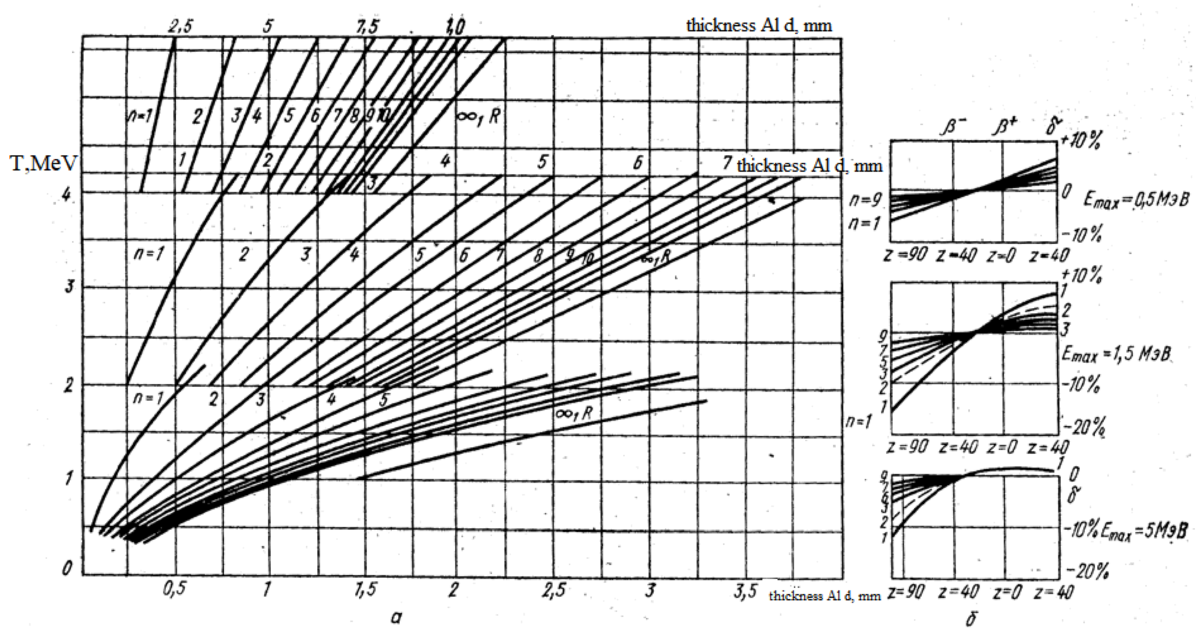

Figure 9. Nomograms

The given nomograms refer to the case when the charge of the β decaying nucleus $Z = 20$. If the charge of the nucleus of the investigated β active substance is not equal to 20 or undergoes β^+ decay rather than β^-, a correction related to the Coulomb interaction between the emitted charged particle and the daughter nucleus in the decay must be introduced. The magnitude of the correction is determined from the panels to the right on Fig. 9.

After determining the maximum electron energy without considering the Coulomb interaction correction, the isotope being measured is identified using decay schemes (provided in the laboratory). To identify the isotope used as a β radioactive source, the results obtained by the first method can also be used.

For a given Z, the correction percentage is found from the nomograms. Then, knowing the thickness of 2^n-fold attenuation $d_n(Z)$, found from the absorption curve, and the correction δ, the thickness of 2^n-fold absorption $d_n(20)$ corresponding to $Z = 20$ is found:

$$d_n(20) = \frac{d_n(Z)}{1 + \delta}$$

Using the found value of dn(20), the upper limit of the β spectrum is determined from the nomograms.

It should be noted that the described methods for determining the upper limit by absorption yield consistent results only in the case of a simple β spectrum, when the decay of the β active nucleus always occurs to the same level (e.g., the ground state) of the product nucleus. In the case of a complex spectrum with comparable intensities of partial spectra, only the upper limit of the partial spectrum with the maximum energy can be determined by finding the practical range of electrons from the absorption curve.

When determining the upper limit of the β spectrum using nomograms (Method II), the results are entered into a table for n from 1 to n_{max} (the maximum possible value of n is related to the accuracy of measuring the absorption curve near the background). After estimating the Z of the β active isotope used as a source of electrons, the correction value is found from the nomograms corresponding to the upper limit of the β spectrum. The average maximum energy value is determined, and the root-mean-square error is calculated.

To submit the work, provide:

1) a graph of the logarithm of the intensity of registered electrons versus absorber thickness;

2) a graph of the intensity of registered electrons versus absorber thickness;

3) determination of the isotope source using formulas for aluminium and nomograms;

4) calculation of the beta decay energy for the found isotope using tabular values (binding energies or masses of related isotopes).